

Multi-Example Search in Rich Information Graphs

***Matteo Lissandrini**, Davide Mottin, Themis Palpanas, Yannis Velegrakis*

A man with a beard is looking through binoculars in a field. The background is a blurred green field under a cloudy sky. A blue banner with white text is overlaid on the bottom half of the image.

What are you looking for?

Search by **a list** of **specifications**

1. A **Movie** and an **Actor**
2. From the **Movie** return the **Director**
3. The **Director** has won an **Award**
4. The **Movie** is adapted from a **Book**
5. From the **Book** return the **Author**

Hard to Specify!

Too many options!
What is the schema?
Which specification is important?



Search by **Example**

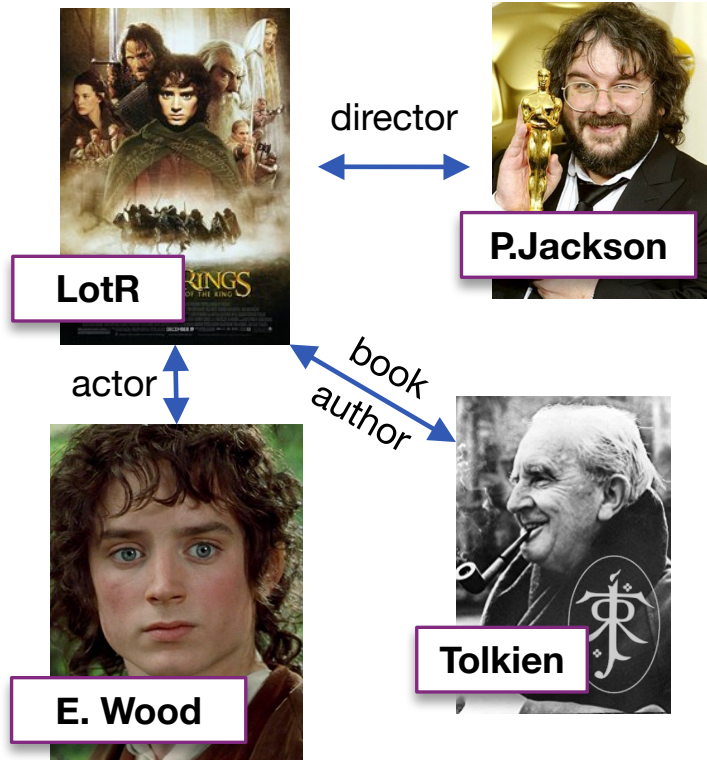
Lord of the Rings

E. Wood

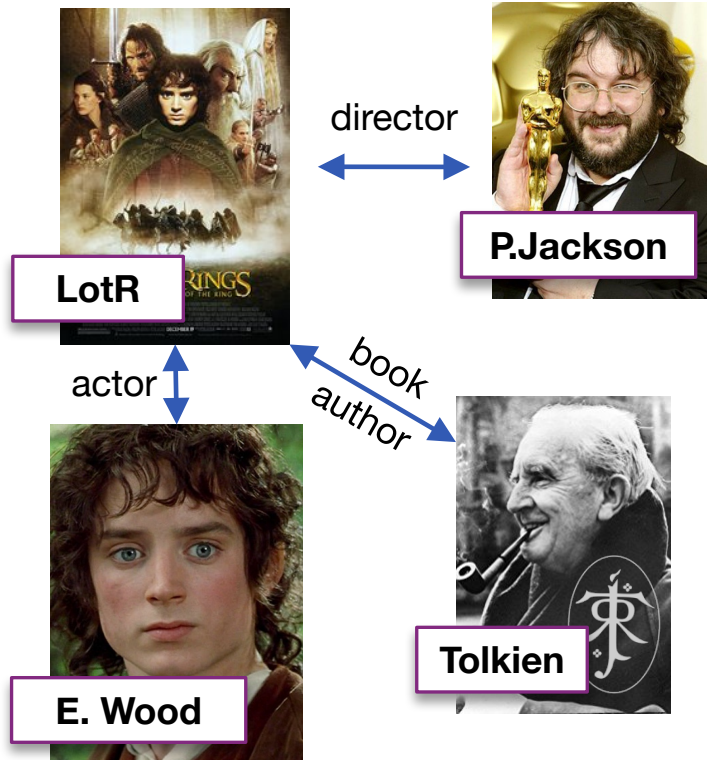
P.Jackson

Tolkien

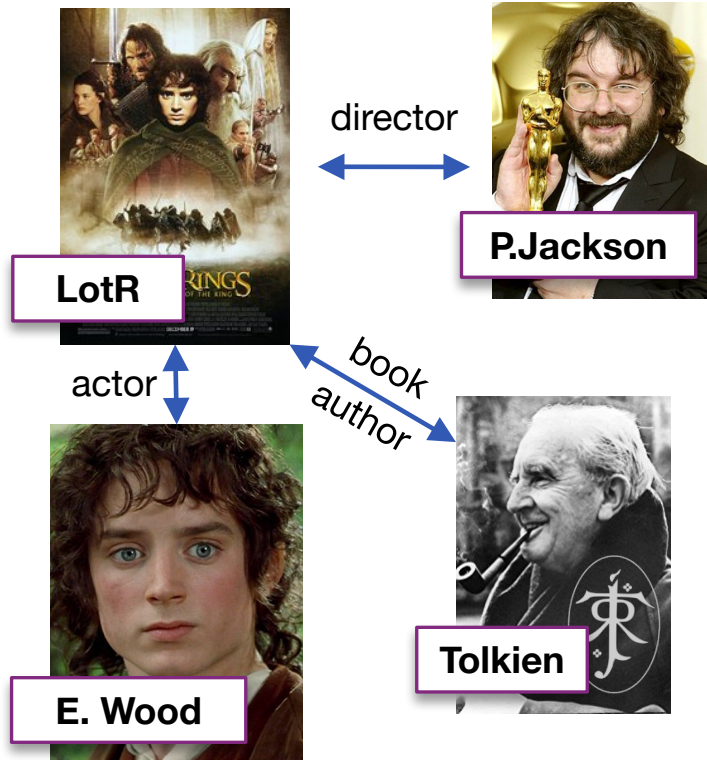
Search by **Example**



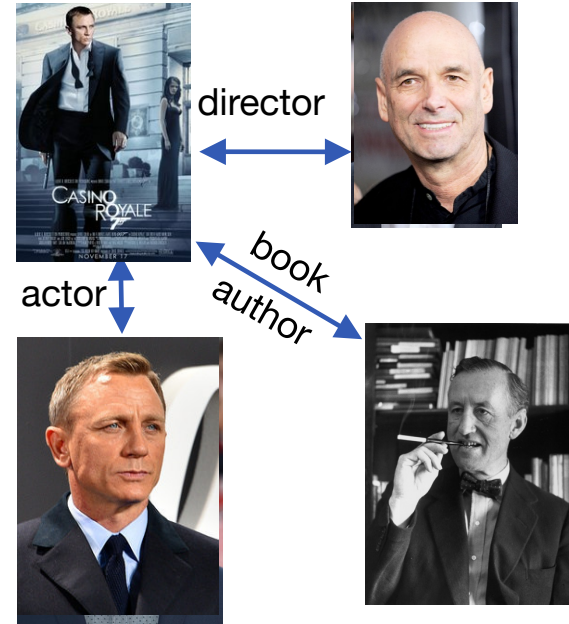
Search by **Example**



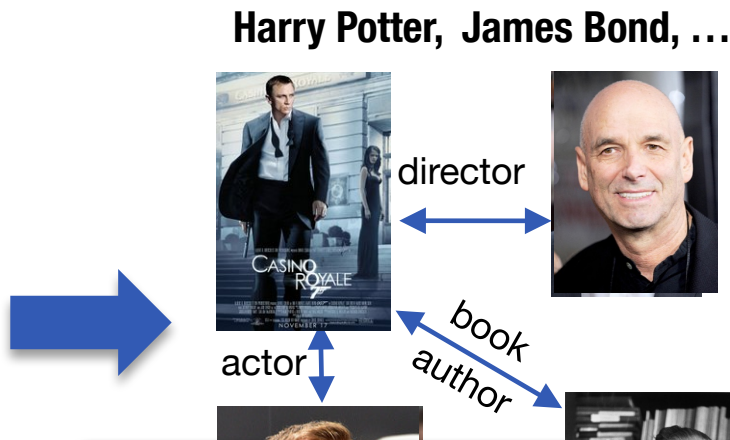
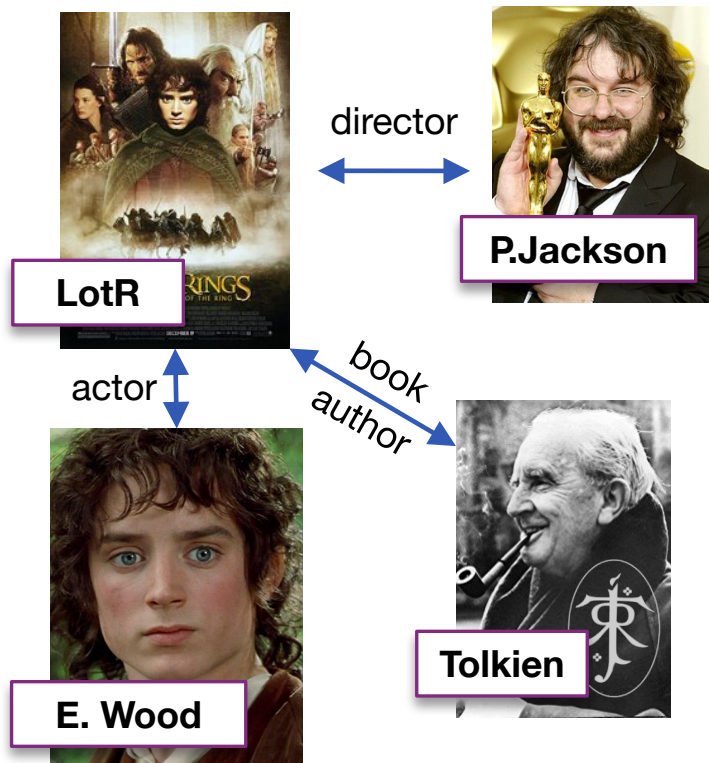
Search by **Example**



Harry Potter, James Bond, ...



Search by **Example**



More Intuitive!

Avoid list of specifications



ONE EXAMPLE IS NOT ENOUGH

When
known examples
are only
Partial Specifications



Search by **Multiple Examples**

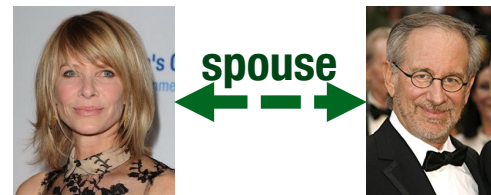
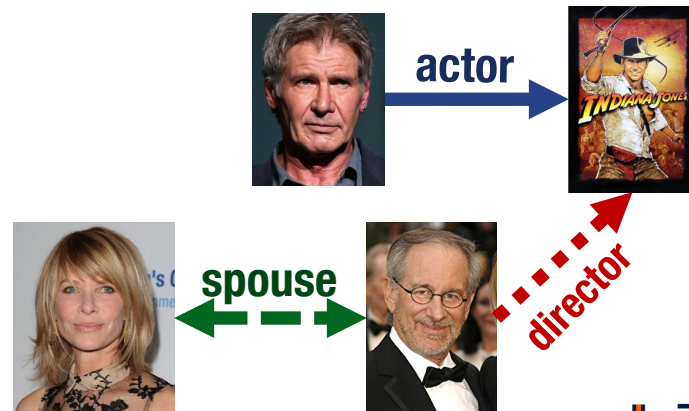
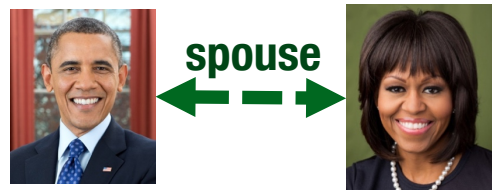
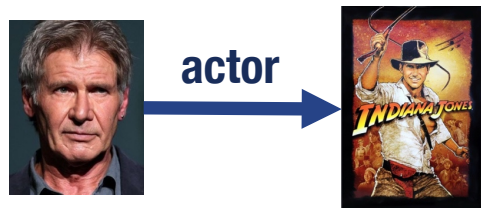


ONE EXAMPLE IS NOT ENOUGH

When
Results have
Different Structures



Search by **Multiple Examples**



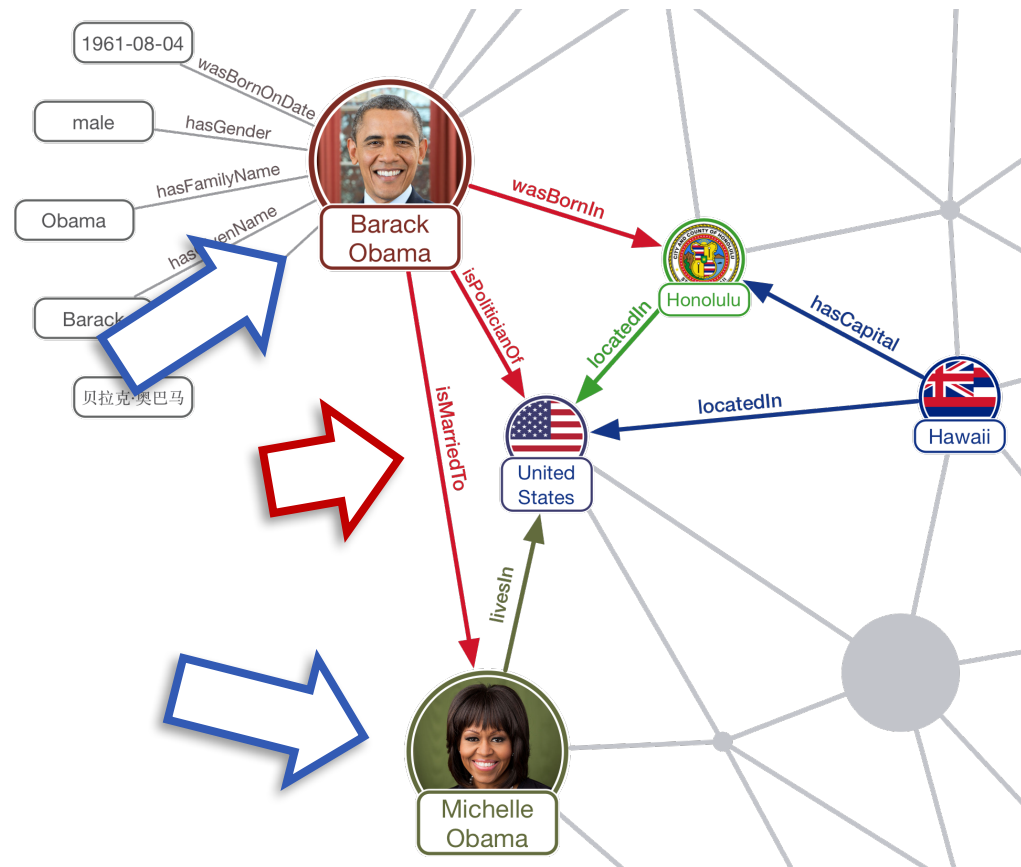
Multi-Example Search

- Multiple Simple Examples
- Each Example describes an Aspect
- Results are Combinations of aspects
- Results have possibly Multiple Structures



INFORMATION GRAPHS

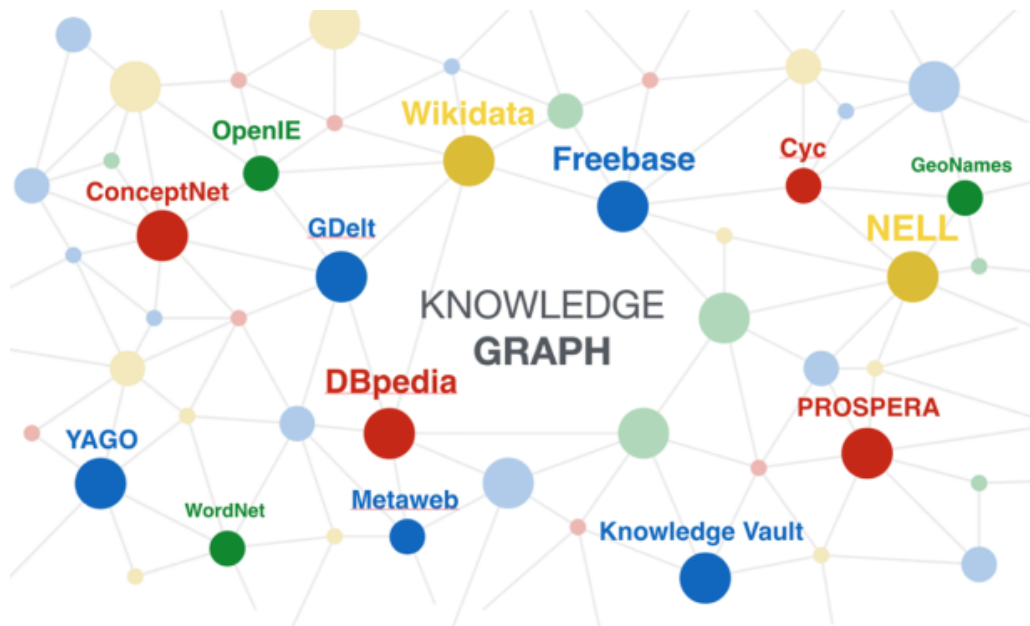
Nodes & Edges



Edge-labelled

Multigraphs

$G: \langle V, E, L, \ell \rangle$



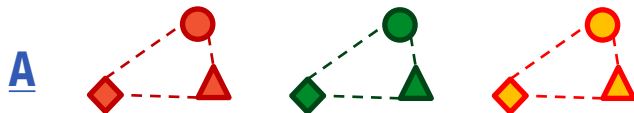
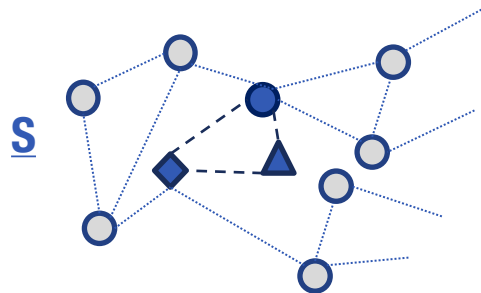
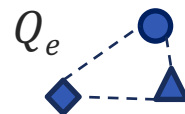
Exemplar Queries

Input: Q_e , an example *element* of interest

Output: set of elements in the *desired result set*

Exemplar Query Evaluation

- match Q_e to sample S in the graph G
- find the set of elements A similar to S given a similarity relation
- [OPTIONAL] return only the top-k subset $A^K \subseteq A$



$$A : \{ a \in D \mid a \approx S \}$$

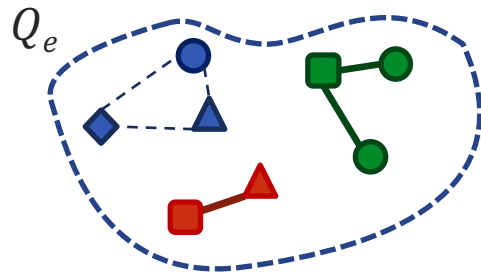
$\approx \rightarrow$ similarity

Multi-Exemplar Queries

Our Problem Formulation:

Input: Q_e , a **set of** example *elements* of interest

Output: set of elements in the *desired result set*



Exemplar Query Evaluation

- **match** **each** $q \in Q_e$ to the **set of samples** $S: \{s_1, s_2, \dots\}$ in the graph G
- **find** the set of **elements** A similar to **each element in** S given a **similarity relation**
- [OPTIONAL] return only the **top-k subset** A^K

$$A : \{ a \in D \mid \forall s \in S . s \approx a \}$$

$\approx \rightarrow$ similarity

(Multi-)Exemplar Queries **on Graphs**

Single Sample $A : \{ a \in D \mid a \approx S \}$

- **Similarity ($\approx$) : graph isomorphism**
 $A : \{ a \text{ subgraph of } G \mid s \text{ isomorphic to } a \}$
- **Challenge : find ALL isomorphic graphs**

Graph Isomorphism is
Transitive and Symmetric!

A is an Equivalence Class



Multiple Samples $A : \{ a \in D \mid \forall s \in S . s \approx a \}$ $s_i, s_j \in S . s_i \neq s_j \rightarrow A = \emptyset$

- **Similarity ($\approx$) : ?Subgraph-Isomorphism**
 $A : \{ a \text{ subgraph of } G \mid \forall s \in S . s \text{ subgraph isomorphic to } a \}$

IS THIS CHARACTERIZATION ENOUGH?

Answers Are Subgraphs
**That Contain Structures Similar
To Each Sample**

Multi-Exemplar **Answers on Graphs**

Graph Similarity ($\approx$) **Subgraph-Isomorphism**

$\{ a \in G \mid \forall s \in S . s \text{ *subgraph isomorphic to* } a \}$

What constitutes a good answer?

**With No Restrictions the Entire Graph
Is Accepted as Answer**

Multi-Exemplar **Answers on Graphs**

Each answer should be
correct, complete and **non redundant**:

Ensure **all Aspects** are present
& **Limit Size** of Answer Graphs

Answers:
WEAKLY CONNECTED SUBGRAPHS
with **NO SUPERFOUS NODES** or **EDGES**

1. Connectedness

$\forall n_1, n_2 \in V_A \exists$ undirected path
that **connects** n_1 to n_2
✓ **CORRECT**

2. Consistency

$\forall n_A \in V_A \rightarrow \exists s \in S, n_s \in V_s$
Such that n_A **maps to** n_s
✓ **COMPLETE** ✓ **NON-REDUNDANT**

CHALLENGE!

To find **Multi-Exemplar** answers

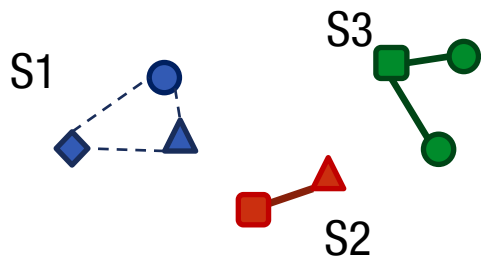
1. Find **ALL isomorphic graphs** to ALL samples
2. Find which samples **combine into one connected** answer

For each sample
needs to perform **Subgraph-Isomorphism Search**

Candidate space = Cartesian Product
for all samples



Search Framework



Multi-exemplar Answering

Input: Database $G : \langle V, E, \ell \rangle$

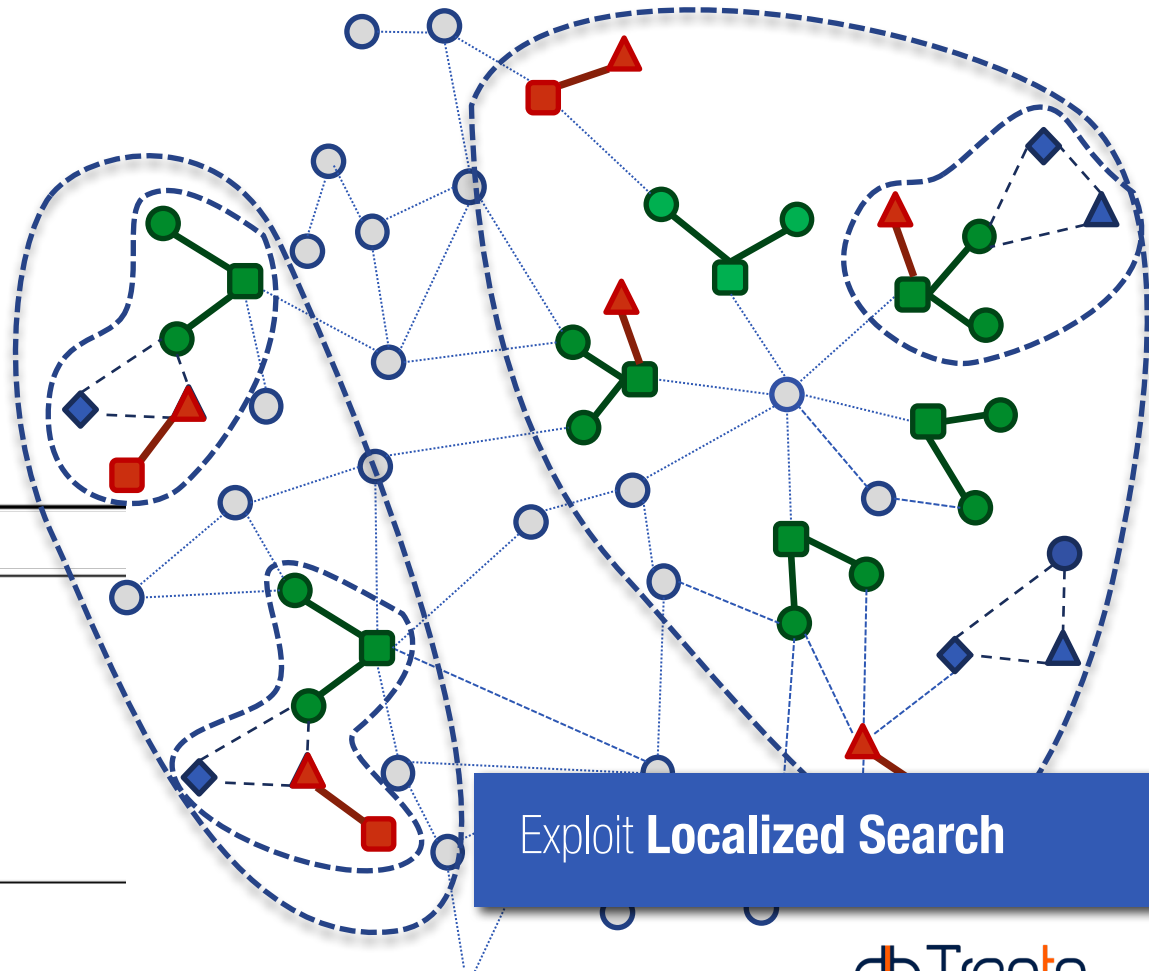
Input: Samples $\mathcal{S} : \langle s_1, \dots, s_m \rangle$

Output: Answers $\mathcal{A}$

1: $\mathcal{G} \leftarrow \text{PARTIAL}(G, \mathcal{S})$

2: $\mathcal{A} \leftarrow \text{SEARCH}(\mathcal{G}, \mathcal{S})$

3: **return** $\mathcal{A}$



Search Framework **Optimizations**

1. Find **CANDIDATE REGIONS**

- a. **Remove Unused Edges**
- b. **Identify SEEDs**
- c. **Expand Around each seed**

2. **SEARCH** within each region

- a. **Avoid Cartesian Product**
- b. **Fast Merge of Partial Aswers**

Multi-exemplar Answering

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Naïve Algorithm

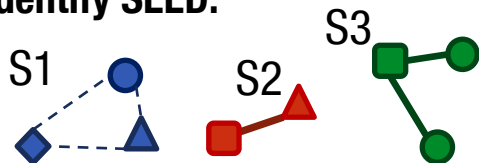
- **1 single** region
- Retrieves **ALL** Isomorphic-subgraphs
- **Hash-JOIN** for Fast merge of Partial Answers

2 Advanced Algorithms

- Fast & Fast+

Find Candidate Regions - Fast

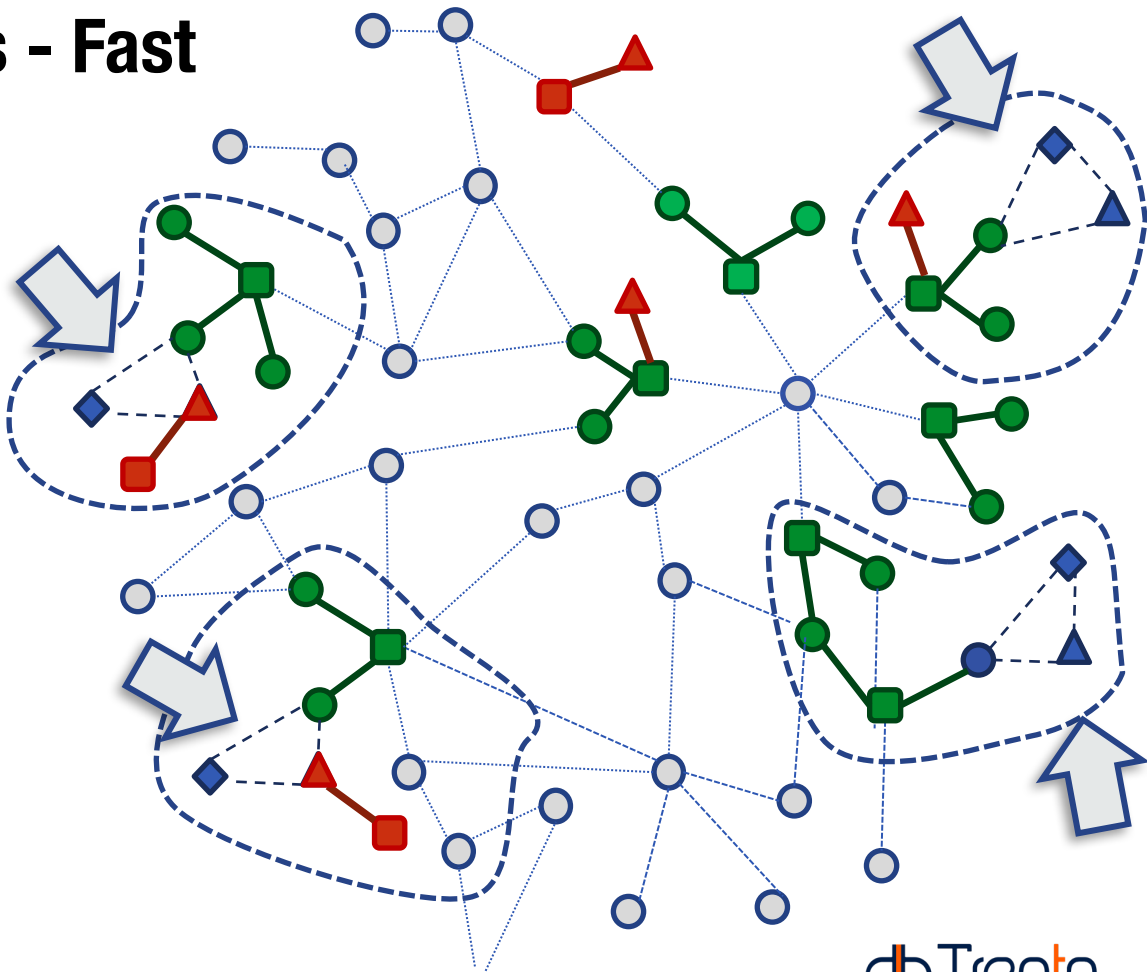
Identify SEED:



Min # of matches S1: 4 S2: 6 S3: 7

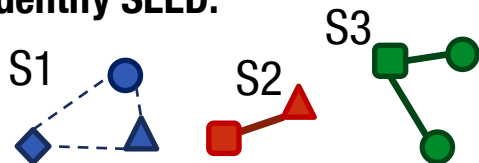
EXPAND around each seed:

Retrieve candidate Regions



Find Candidate Regions - Fast

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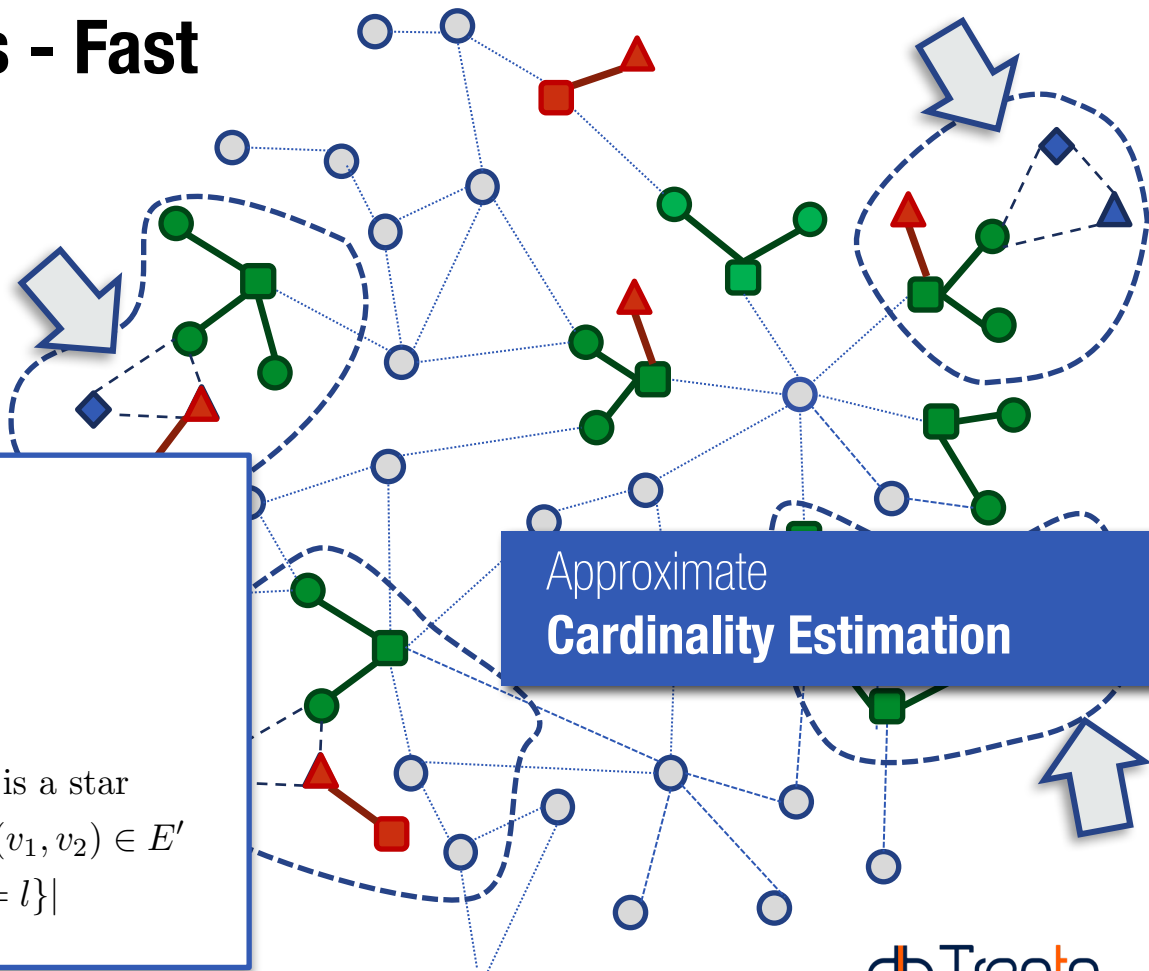


Min # of matches S1: 4 S2: 6 S3: 7

Precomputed Statistics:

- Label frequency
- Label Pair frequency
- Star cardinality

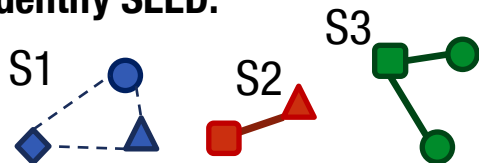
$$\mathbb{I}_{star}(l, c) = |\{G' \subseteq G \mid G' : \langle V', E', \ell \rangle \text{ is a star} \\ \wedge |E'| = c \wedge \exists (v_1, v_2) \in E' \\ \text{s.t. } \ell(v_1, v_2) = l\}|$$



Approximate
Cardinality Estimation

Find Candidate Regions - Fast

Identify SEED:



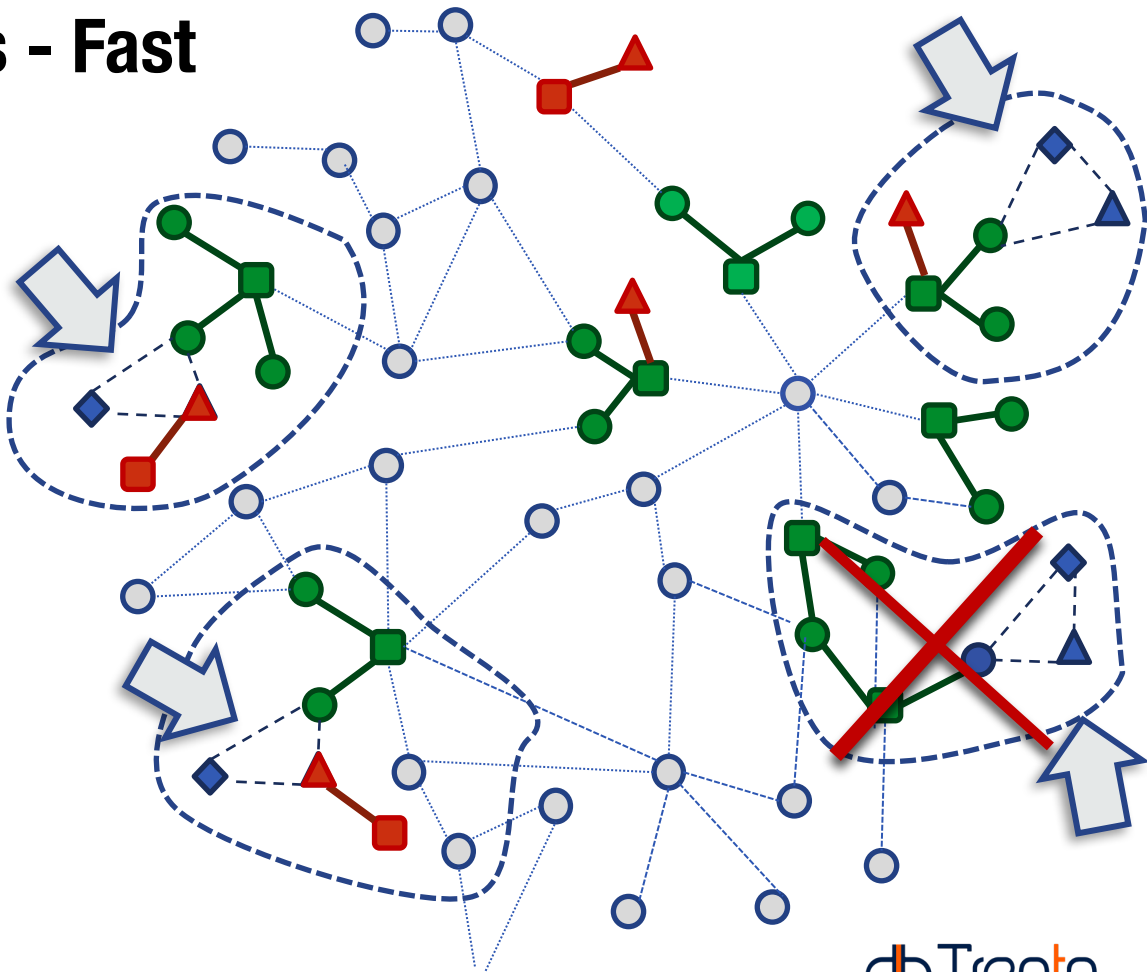
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EXPAND around each seed:

Retrieve candidate Regions

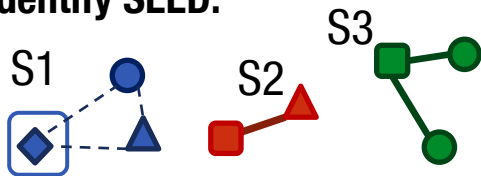
Seed search
requires **Isomorphic-Search**

Some Regions
do **not** contain all structures



Find Candidate Regions – Fast+

Identify SEED:



With cardinality Estimation

Select SINGLE NODE

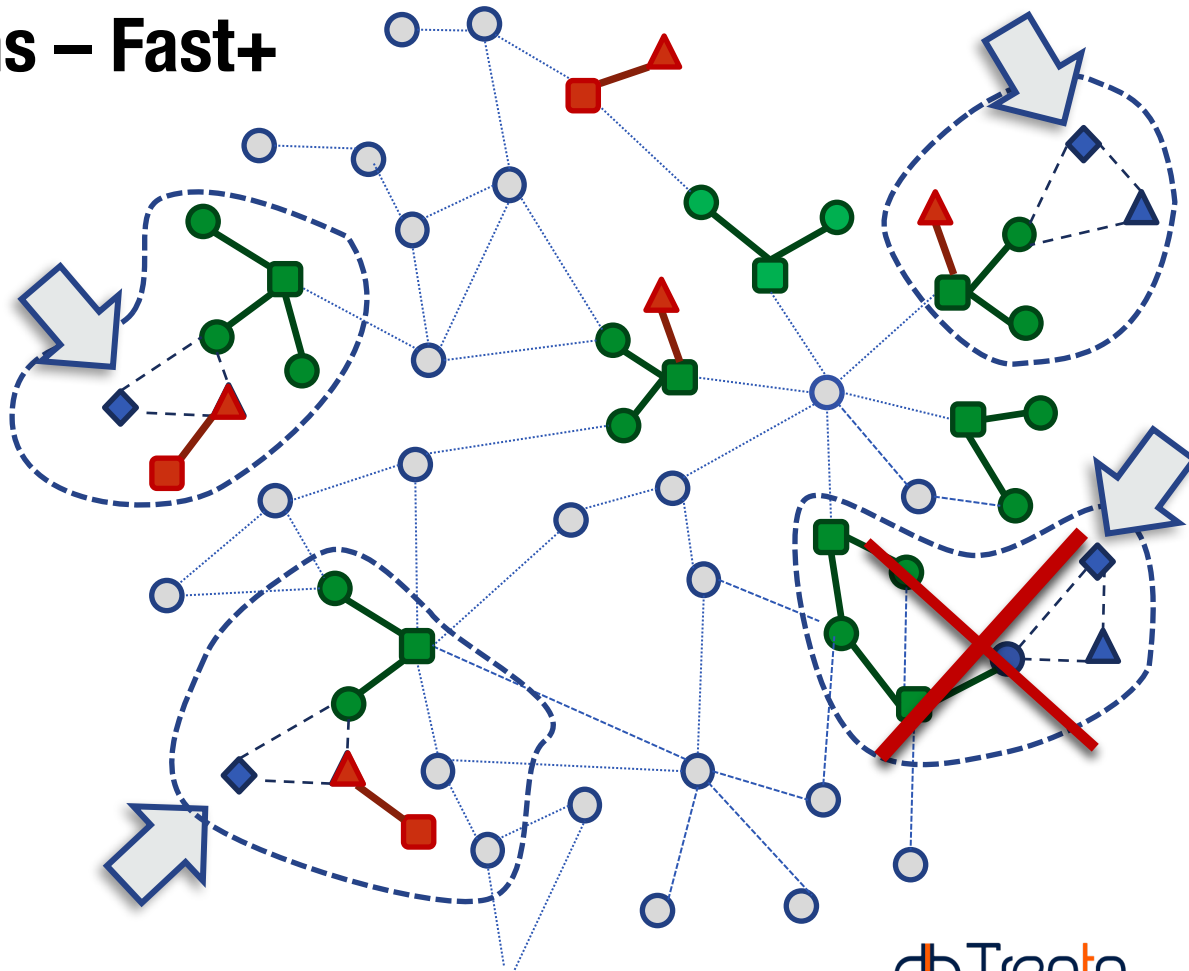
With bitset-mapping

EXPAND around each seed:

Retrieve candidate Regions

DISCARD incomplete regions

With bitset-mapping & before ISO-search



Fast Pruning with Bit-Vectors

	d=1				
	actor	director	spouse	author	...
<u>B</u> arack <u>O</u> bama (S2)	-	-	1	-	
<u>Q</u> uentin <u>T</u> arantino (S3)	-	1	-	-	
<u>S</u> teven <u>S</u> pielberg (A1)	-	1	1	-	
<u>K</u> ate <u>C</u> apshaw (A2)	-	-	1	-	
BO $\vee$ QT	-	1	1	-	(union)
(BO $\vee$ QT) $\wedge$ $\overline{\mathbf{KC}}$	-	1	-	-	($\neq 0$)
(BO $\vee$ QT) $\wedge$ $\overline{\mathbf{SS}}$	-	-	-	-	($= 0$)

Example:

Detect JOIN-Node



Top-K

Scoring Function $\rho(A) = \frac{1}{|V_A|} \sum_{v \in V_A} w(v)$

Weight Function for Nodes 

Multi-exemplar Answering

Input: Database $G : \langle V, E, \ell \rangle$
Input: Samples $\mathcal{S} : \langle s_1, \dots, s_m \rangle$
Output: Answers $\mathcal{A}$

- 1: $\mathcal{G} \leftarrow \text{PARTIAL}(G, \mathcal{S})$
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- 3: **return** $\mathcal{A}$

Theorem 2. *Given the set of graph samples $\mathcal{S}$, and answers, A_1 and A_2 , $\forall s \in \mathcal{S}. s \subseteq A_i$, via the isomorphism function $\mu_{A_i}^s$, the node weight function w , and the ranking ρ (Equation 1). It holds:*

$$\bar{\rho}(A_2) = \frac{\sum_s^{\mathcal{S}} \sum_v^{V_s} w(\mu_{A_2}^s(v))}{\max_{s \in \mathcal{S}} |V_s|} < \rho(A_1) \rightarrow \rho(A_2) < \rho(A_1) \quad (4)$$

**Skip Regions that will not produce answers
with score High Enough**

Top-K

Scoring Function $\rho(A) = \frac{1}{|V_A|} \sum_{v \in V_A} w(v)$

Weight Function for Nodes

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Multi-exemplar Answering

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3: **return** $\mathcal{A}$

1. Sort regions (output of Partial)
2. Estimate **Upper Bound Score** for each
3. Find Top-K & prune

Experimental Evaluation

3 Algorithms:

- a) **Naïve**
- b) **Fast** (iso-graphs as seeds)
- c) **Fast+** (nodes as seeds)

2 Large Real Datasets:

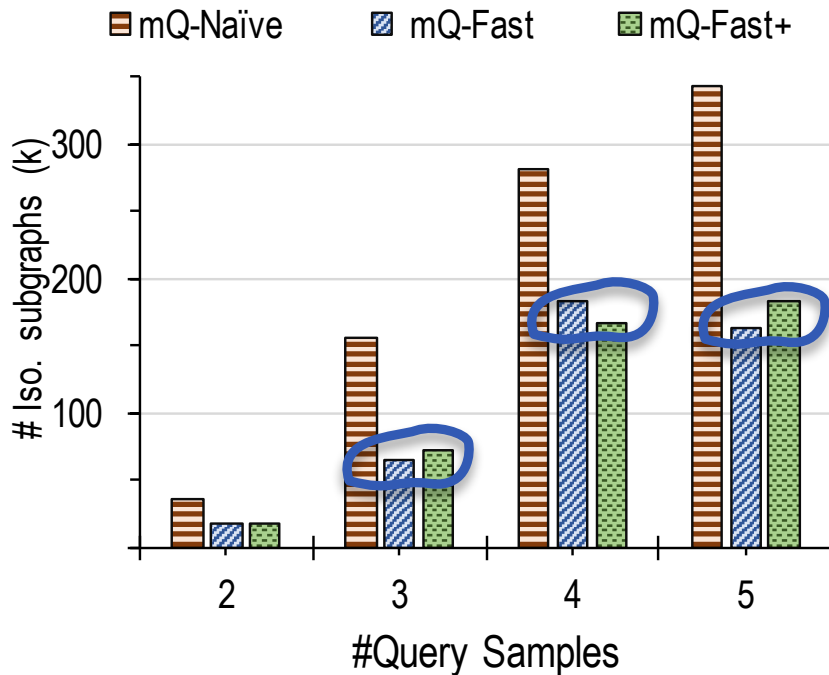
- a) **YAGO +16.7M Edges**
- b) **Freebase +300M Edges**

Tests: 100 Queries 2-5 Samples

- **Count isomorphic computations**
- **Running Time**

Evaluation Results

Datasets: Freebase (300M Edges) Yago (16.7M edges)
100 queries, from 2 to 5 samples

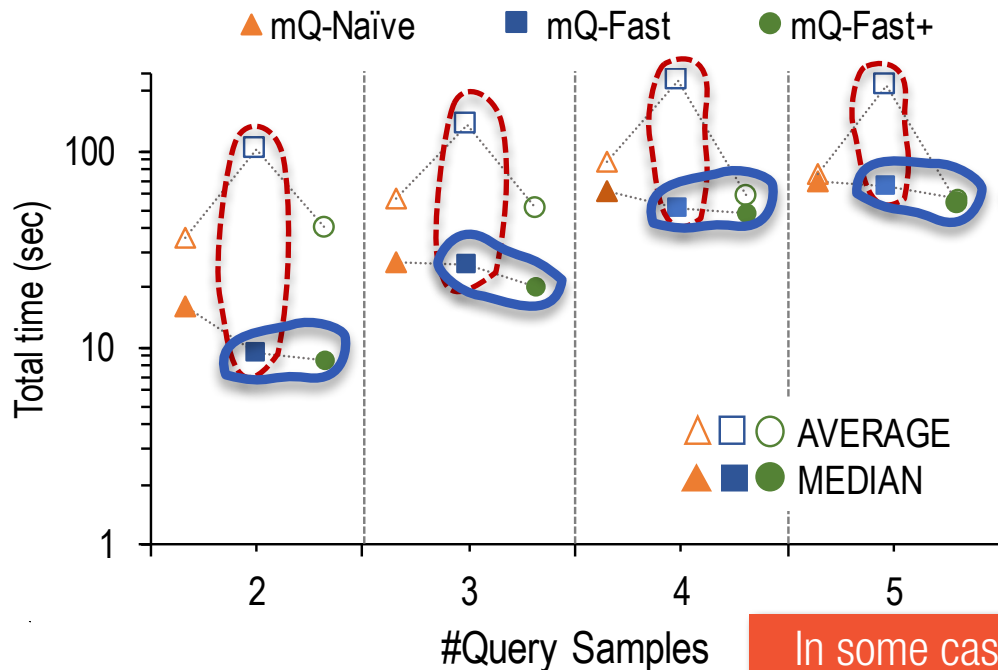


**40-60%
Reduction in Iso-Search
Computations!**

Evaluation Results

Datasets: Freebase (300M Edges) Yago (16.7M edges)
100 queries, from 2 to 5 samples

Running
Times



**Faster 70% of queries
Saves up to
25secs on AVG**

In some cases the “**Fast**” algorithm
wastes computations

Conclusions

Search via **Multiple-Examples**

Find **structures similar to a SET of input** examples.

Output:

Composite Results

Containing characteristics from
each Example

Useful When:

a **Complete Example** with **all** the desired characteristics
is not known.

Characteristics **combine** in **Multiple** ways.

Optimizations:

1. Exploit **Localized Search**
2. **Bitset Pruning**
3. **Cardinality Estimation**
4. **Top-K Optimizations**

Thank You!
Questions?

p.s.
**Now you
can hire me!**

Localized Search

Is less effective

With a Dense Graph!



Thank you!
Questions?

There is more . . .

Search Framework **Optimizations (bis)**

Algorithm 1 mQ-Naive

```
1: function PARTIAL( $G, \mathcal{S}$ )
2:   return  $\mathcal{G} : \{G\}$   $\triangleright$  Selectively pruned based on  $\mathcal{S}$ 
3: function SEARCH( $\mathcal{G}, \mathcal{S}$ )
4:    $\mathcal{A} \leftarrow \emptyset$ 
5:   for each  $G \in \mathcal{G}$  do
6:     for each  $s_i \in \mathcal{S}$  do  $\triangleright$  Find isomorphic subgraphs
7:        $\tilde{A}_i \leftarrow \{A \subseteq G | s_i \approx A\}$ 
8:        $C \leftarrow \arg \min_{\tilde{A}_i \in \{\tilde{A}_1, \dots, \tilde{A}_{|\mathcal{S}|}\}} |\tilde{A}_i|$ 
9:       while  $C \neq \emptyset$  do
10:         $c \leftarrow \text{REMOVEONE}(C)$ 
11:        if  $\forall s \in \mathcal{S}. s \sqsubseteq c$  then  $\triangleright$  Check sample in  $c$ 
12:           $\mathcal{A} \leftarrow \mathcal{A} \cup \{c\}$ 
13:        else  $C \leftarrow C \cup \text{CONNECT}(c, \mathcal{S}, \langle \tilde{A}_1, \dots, \tilde{A}_{|\mathcal{S}|} \rangle)$ 
14:   return  $\mathcal{A}$ 
```

Algorithm 2 CONNECT⁺

Input: Candidate $c : \langle V_c, E_c, \ell_c \rangle$; HashTable $\mathbb{H}$

Output: Expanded candidates C^+

```
1:  $C^+ \leftarrow \emptyset$ 
    $\triangleright$  Find candidate nodes contained in some answer
2: for each  $n \in V_c$  do
3:   for each  $\tilde{A}_i \in \mathbb{H}(n)$  s.t.  $s_i \not\sqsubseteq c$  do
4:      $C^+ \leftarrow C^+ \cup \text{MERGE}(c, \tilde{A}_i)$ 
5: return  $C^+$ 
```

Multi-exemplar Answering

Input: Database $G : \langle V, E, \ell \rangle$

Input: Samples $\mathcal{S} : \langle s_1, \dots, s_m \rangle$

Output: Answers $\mathcal{A}$

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Naïve Algorithm

- **1 single** region
- Retrieves **ALL** Isomorphic-subgraphs
- **Hash-JOIN** for Fast merge of Partial Answers

2 Advanced Algorithms

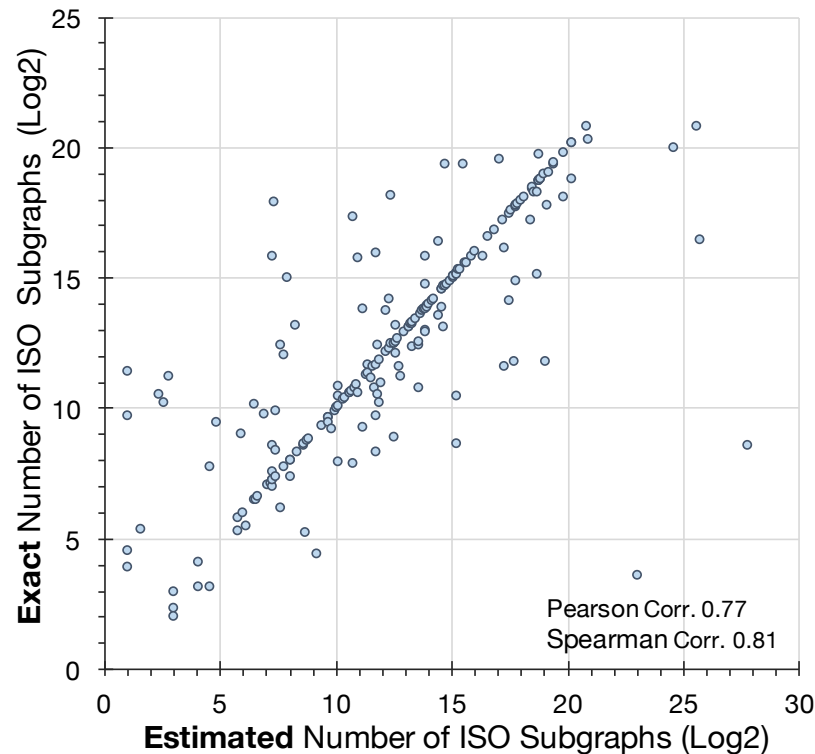
- Fast & Fast+

Cardinality Estimation

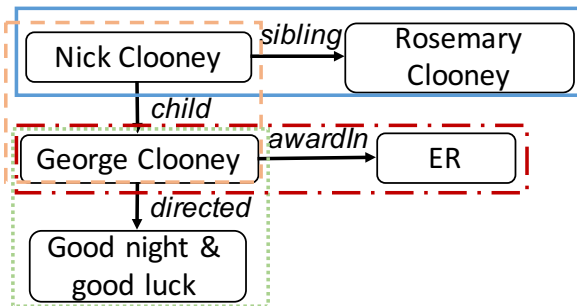
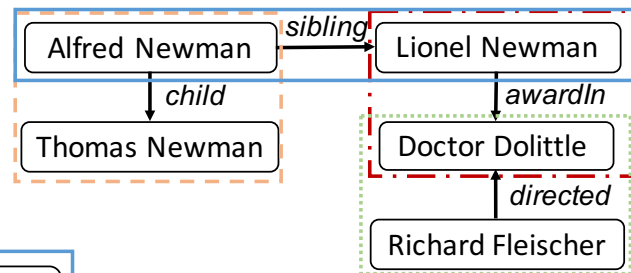
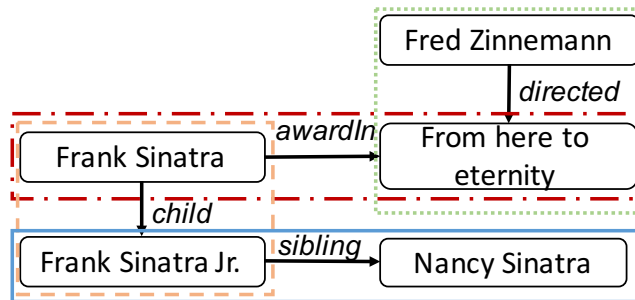
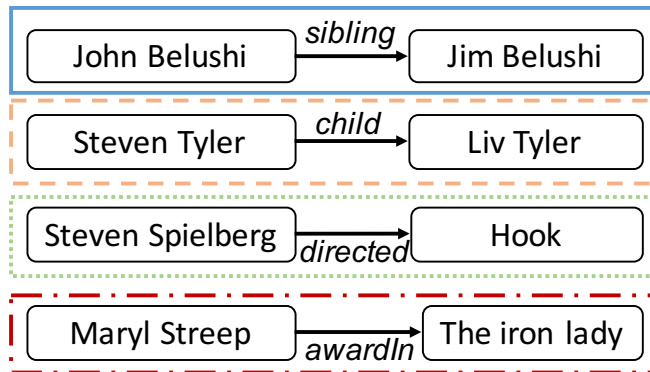
Precomputed Statistics:

- **Label frequency**
- **Label Pair frequency**
- **Star cardinality**

$$\mathbb{I}_{star}(l, c) = |\{G' \subseteq G \mid G' : \langle V', E', \ell \rangle \text{ is a star} \\ \wedge |E'| = c \wedge \exists (v_1, v_2) \in E' \\ \text{s.t. } \ell(v_1, v_2) = l\}|$$

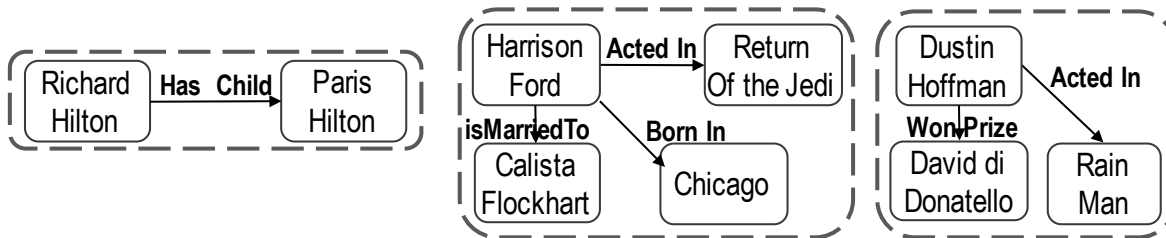


Example Query/Answers



Example Query/Answers

Examples



Answers

